ÉRETTSÉGI VIZSGA ● 2007. május 8.

MATEMATIKA
ANGOL NYELVEN
MATHEMATICS

EMELT SZINTŰ ÍRÁSBELI
ÉRETTSÉGI VIZSGA
ADVANCED LEVEL
WRITTEN EXAM

JAVÍTÁSI-ÉRTÉKELÉSI
ÚTMUTATÓ
KEY AND GUIDE FOR
EVALUATION

OKTATÁSI ÉS KULTURÁLIS
MINISZTÉRIUM
MINISTRY OF EDUCATION
AND CULTURE

Important Information

Formal requirements:

1. The papers must be assessed in **pen and of different colour** than the one used by the candidates. Errors and flaws should be indicated according to ordinary teaching practice.
2. The first one among the shaded rectangles next to each question contains the maximal score for that question. The **score** given by the examiner should be entered into the other **rectangle**.
3. **In case of correct solutions**, it is enough to enter the maximal score into the corresponding rectangle.
4. In case of faulty or incomplete solutions, please indicate the corresponding **partial scores** within the body of the paper.

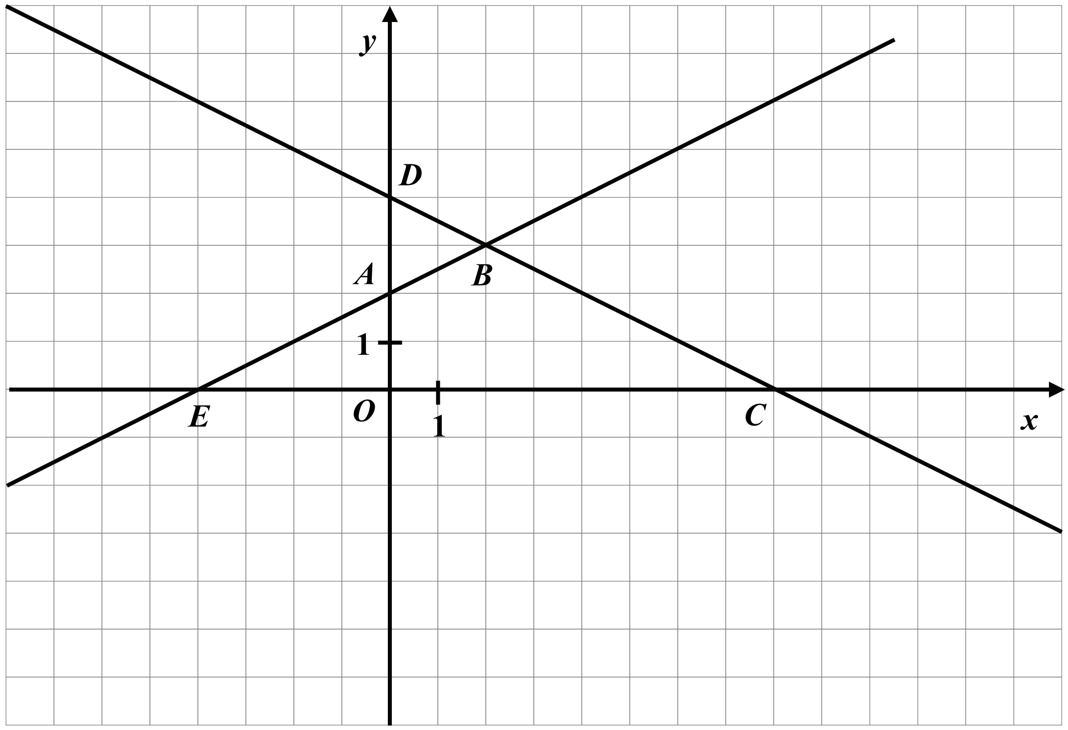
Substantial requirements:

1. In case of some problems there are more than one marking schemes given. However, if you happen to come across with some **solution different** from those outlined here, please check the parts equivalent to those in the solution provided here and do your marking accordingly.
2. The scores in this assessment **can be split further**. Remember, however, that the number of points given for any item can be an integer number only.
3. If the answer is correct and the argument is clearly valid then the maximal score can be given even if the actual solution is **less detailed** than that in this booklet.
4. If there is a **calculation error** or any other flaw in the solution, then the score should be deducted for the actual item only where the error has occured. If the candidate is going on working with the faulty intermediate result and the problem has not suffered substantial damage due to the error, the subsequent partial scores should still be given.
5. If there is a **fatal error** within an item (these are separated by double lines in this booklet), then even formally correct steps should not be given any points, whatsoever. However, if the wrong result obtained by the invalid argument is used correctly throughout the subsequent steps, the candidate should be given the maximal score for the remaing parts, unless the problem has been changed essentially due to the error.
6. If an **additional remark** or a **measuring unit** occurs in brackets in this booklet, the solution is complete even if the candidate does not mention it.
7. If there are more than one correct attempts to solve a problem, it is the **one indicated by the candidate that can be marked**.
8. You should **not give any bonus points** (points beyond the maximal score for a solution or for some part of a solution).
9. You **should not reduce** the score for erroneous calculations or steps unless its results are actually used by the candidate in the course of the solution.
10. **There are only 4 questions to be marked out of the 5 ones in part II. of this exam paper.** Hopefully, the candidate has entered the number of the question not to be marked in the square provided for this. Accordingly, this question should not be assessed even if there is some kind of solution contained in the paper. Should there be any ambiguity about the student's request with respect to the question not to be considered, it is the last one in this problem set, by default, that should not be marked.

I.

1.		
Rewriting the first equation by virtue of the laws of logarithms yields $\log_2 \frac{2x+y}{x-1{,}5y} = \log_2 4$.	2 points	
By monotonicity one gets	1 point	
$\frac{2x+y}{x-1{,}5y} = 4$, and thus, after simplifying one gets $7y = 2x$, that is $x = 3.5y$.	1 point	
Rewriting the first equation by virtue of the laws of logarithms yields $\log_3 (x-y)(x+y) = \log_3 45$.	2 points	
Using the corresponing identity and the monotonicity of the logarithm function yields $x^2 - y^2 = 45$.	1 point	
Substituting $x = 3.5y$ and simplifying: $y^2 = 4$,	1 point	
therefore $y = 2$ or $y = -2$.	1 point	*This point cannot be given if the candidate finds $y = 2$ only.*
If y is negative then so is the corresponding value of x and thus these numbers do not satisfy the equation.	1 point	
The only solution is the pair $x = 7$ and $y = 2$. These numbers satisfy both equations.	1 point	
Total:	**11 points**	

2. a)



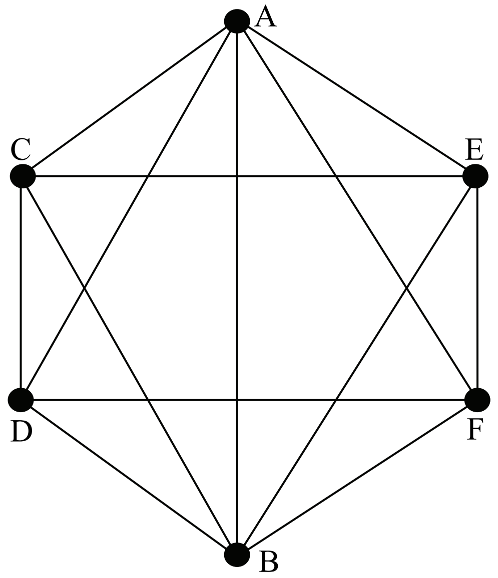
Correct diagram.	2 points	
Total:	**2 points**	

b)

The vertices of the convex quadrilateral are *A, B, C, O*, where *B* is the intersection of the two straight lines: *B*(2, 3).	1 point	
The area of the quadrilateral can be computed, for example, by subtracting the area of the triangle *ABD* from that of the triangle *DOC*. The area of the right triangle *DOC* is $\frac{OC \cdot OD}{2} = \frac{8 \cdot 4}{2} = 16$.	2 points	
In the triangle *ABD* clearly *AD* = 2 and the length of the height dropped from *B* is equal to the abscissa (first coordinate) of *B*: since the latter is 2, the area of the triangle *ABD* is equal to 2.	2 points	
The area of the convex quadrilateral *ABCO* is equal to 16 − 2 = 14 (area units).	1 point	
Total:	**6 points**	

c)		
The vertices of the concave quadrilateral are *E, C, D, A*. The lengts of the sides are $EC = 12$; $CD = \sqrt{80}$; $DA = 2$; $AE = \sqrt{20}$; $ED = 4\sqrt{2}$; $CA = \sqrt{68}$.	4 points	*The length of each side is worth 1 point.*
The perimeter is $k_1 = EC + CD + DA + AE =$ $= 12 + \sqrt{80} + 2 + \sqrt{20} = 14 + 6\sqrt{5}$ ($\approx$ 27,42); $k_2 = ED + DC + AC + AE = 4\sqrt{2} + 6\sqrt{5} + 2\sqrt{17}$ ($\approx$ 27,32); $k_3 = ED + AD + AC + EC = 14 + 4\sqrt{2} + 2\sqrt{17}$ ($\approx$ 27,91).	1 point	
Total:	**5 points**	The candidate receives 5 points if s/he has calculated correctly the perimeter of at least one of the three possible concave quadrangles.

3. a)

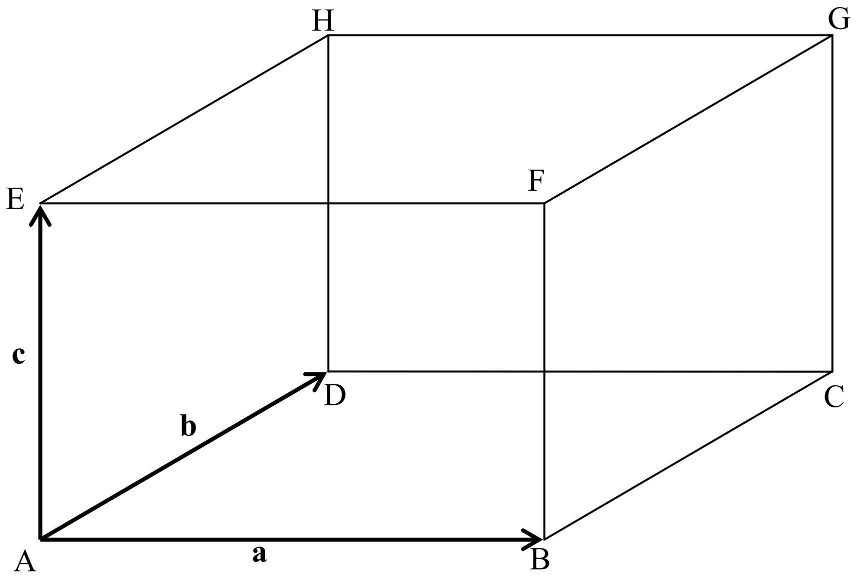


There should be 6 vertices in the correct diagram,	1 point	
two of them are of degree five (*A* and *B*)	1 point	
and four of them are of degree four (*C, D, E, F*).	2 points	
Total:	**4 points**	*There are two edges missing from the complete 6-point graph. The subgraph formed by these two edges in the complementary graph is not connected.*
b)		
Summing the degrees in this graph we get the double of the number of handshakes.	1 point	
The sum of the degrees is equal to 26.	1 point	
The travellers were greeting each other by exchanging 13 handshakes, altogether.	1 point	
Total:	**3 points**	*These 3 marks may also be given if the candidate counts the edges in the correct diagram.*

c) 1st solution		
Chose first a roommate for scientist A. There are five ways to do that.	1 point	
Having selected the roomate for A and chosing one – say C – from the remaining group of four there are three ways to select C's roomate.	2 points	
Having already taken care of two rooms there is just one way for the remaining two scientist to move into the third room.	1 point	
Since the rooms are not distinguished, there are $5 \cdot 3 \cdot 1 = 15$ arrangements altogether.	2 points	
Total:	**6 points**	*These 6 points are also due if the candidate compiles an organized list containing the 15 arrangements. If the list is deficient but still well organized, then no more than 4 points can be given.*

c) 2nd solution		
There are $\binom{6}{2}$ ways to select two scientists.	2 points	
There are $\binom{4}{2}$ ways two select another two from the remaining four scientists.	1 point	
Therefore, there are $\binom{6}{2} \cdot \binom{4}{2}$ ways to split them into three groups of two members each.	1 point	
There are 3! ways of assigning these groups to the three rooms and thus there are $\dfrac{\binom{6}{2} \cdot \binom{4}{2}}{3!} = 15$ arrangements altogether in the rooms.	2 points	
Total:	**6 points**	

4. a)



The sum of the seven vectors is $\overrightarrow{AP} = (\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{AE}) + \left(\overrightarrow{AC} + \overrightarrow{AF} + \overrightarrow{AH}\right) + \overrightarrow{AG}$. Expressing the vectors, respectively, on the r.h.s. in terms of the edge vectors **a**, **b** and **c** $\overrightarrow{AP} = (\mathbf{a} + \mathbf{b} + \mathbf{c}) + (\mathbf{a} + \mathbf{b}) + (\mathbf{a} + \mathbf{c}) + (\mathbf{b} + \mathbf{c}) + (\mathbf{a} + \mathbf{b} + \mathbf{c})$.	1 point	*This point is due for any correct representation of the vector* $\overrightarrow{AP}$.
Collecting the equal terms one gets $\overrightarrow{AP} = 4(\mathbf{a} + \mathbf{b} + \mathbf{c})$.	1 point	
Total:	**2 points**	
b)		
Since $\overrightarrow{AP} = 4\overrightarrow{AG}$, the modulus of $\overrightarrow{AP}$ is equal to four times the length of the space diagonal AG.	1 point	
By Pithagoras' theorem $AG^2 = AB^2 + BC^2 + CG^2 = 10^2 + 8^2 + 6^2 = 200$, $AG = \sqrt{200} = 10\sqrt{2}$.	1 point	
$AP = 4AG = 40\sqrt{2}$ (≈ 56.57).	1 point	
Total:	**3 points**	
c)		
Since $\overrightarrow{AP} = 4\overrightarrow{AG}$, the angle of the vectors $\overrightarrow{AP}$ and $\overrightarrow{AE}$ is equal to that of the vectors $\overrightarrow{AG}$ and $\overrightarrow{AE}$. This is angle A in the right triangle AEG; denote it by α.	1 point	
Since $AE = 6$ and $AG = 10\sqrt{2}$, $\cos\alpha = \frac{AE}{AG} = \frac{6}{10\sqrt{2}} \approx 0.4243$,	1 point	
and hence $\alpha \approx 64.9°$.	1 point	
Total:	**3 points**	

d)		
The position vector $\overrightarrow{AS}$ of the centroid S of the triangle HFC is equal to one third of the sum of the position vectors of the vertices.	2 points	
$\overrightarrow{AS} = \frac{\overrightarrow{AH} + \overrightarrow{AF} + \overrightarrow{AC}}{3} =$ $= \frac{(\mathbf{b}+\mathbf{c})+(\mathbf{a}+\mathbf{c})+(\mathbf{a}+\mathbf{b})}{3} =$ $= \frac{2}{3}(\mathbf{a}+\mathbf{b}+\mathbf{c})$,	1 point	
and thus $\overrightarrow{AS} = \frac{2}{3}\overrightarrow{AG}$.	1 point	
$\overrightarrow{AS} \cdot \overrightarrow{AP} = (\frac{2}{3}\overrightarrow{AG})\cdot(4\overrightarrow{AG}) =$ $= \frac{8}{3}AG^2 = \frac{8}{3}\cdot 200 = \frac{1600}{3}\ (\approx 533.3)$.	2 points	
Total:	**6 points**	

II.

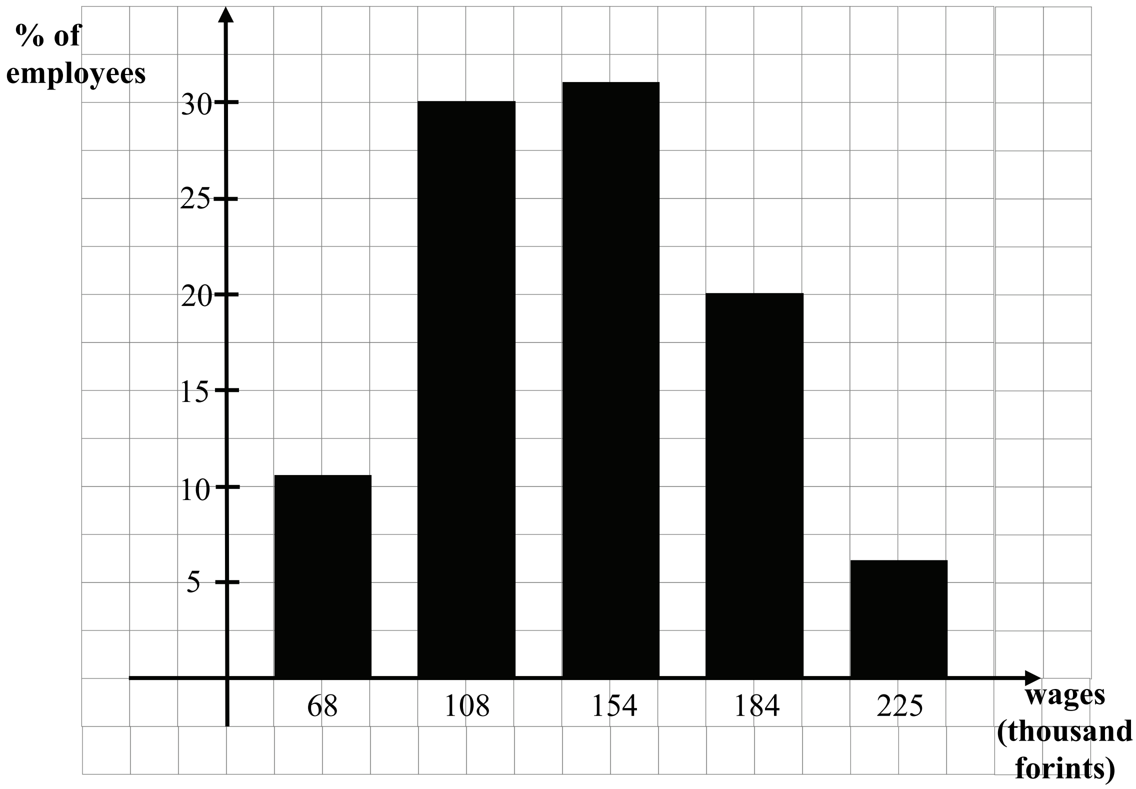
5.		
Factorising the denominators $\frac{x}{(x-2)(x+2)} + \frac{p}{x(x+2)} + \frac{1}{x(2-x)} = 0$	2 points	
Since the denominators cannot be equal to 0, the forbidden values for x are −2; 0; 2.	1 point	
Multiplying by the common denominator $(x-2)x(x+2)$ and rearranging according to the decreasing powers of x one gets the equation $x^2+(p-1)x-2(p+1)=0$.	1 point	
By the quadratic formula $x_{1,2}=\frac{1-p\pm\sqrt{p^2+6p+9}}{2}$,	1 point	
that is $x_{1;2}=\frac{1-p\pm\lvert p+3\rvert}{2}$.	1 point	*This point is due for recognizing the complete square.*
$x_1=2$ and $x_2=-(p+1)$.	2 points	
It is impossible for the given equation to have two distinct solutions since one of these roots, namely $x_1=2$ is a forbidden value for *x*. Therefore, there can be at most one solution, $x_2=-(p+1)$.	2 points	
There is no solution if x_2 is equal to any one of the excluded values, −2; 0; 2.	2 points	
$x_2=-2$, if $p=1$; $x_2=0$, if $p=-1$; $x_2=2$, if $p=-3$.	3 points	
Therefore the equation has no real roots if *p* assumes one of the values −3; −1 or 1.	1 point	
Total:	**16 points**	

6. a)		
By the geometric mean property we have $p^2=4c$.	1 point	*If the candidate is familiar with the notions of arithmetic and geometric progression and makes correct observations but somewhere gets stuck (most probably because it uses too many variables...), at most 2 points may be given.*
By the arithmetic mean property we have $2c=p+40$.	1 point	
Substituting the value of $2c$ into the first equation and collecting the terms one gets $p^2-2p-80=0$,	1 point	
hence $p_1=10$ and $p_2=-8$.	1 point	
Since there is no solution for the negative root, Danny has counted 10 big red fish and 25 small striped fish altogether.	1 point	
Total:	**5 points**	

b)		
The growth rate of the fish is 20 % each month and thus their amount should be scaled up monthly by 1.2 .	1 point	
If Danny sold x % of the fish every other month then their amount should be multiplied bimonthly by $\left(1-\frac{x}{100}\right)=q$.	1 point	
Therefore, the bimonthly scale factor is equal to $1.2^2\cdot q=1.44\cdot q$.	1 point	
This yields the equation $100\cdot(1{,}44q)^{12}=252$.	1 point	
Solving this equation one gets q = 0,75.	2 points	
Hence Danny was left with the 75 % of his fish and thus he sold 25 % of them every other month.	1 point	
Total:	**7 points**	
c)		
There are $\binom{20}{8}$ equally probable ways of selecting 8 fish out of 20.	1 point	*Correct values of the binomial coefficients should be accepted even if the candidate does not indicate their actual calculations.(Calculators or a formula sheet might have been of help). However, if the actual value of the probability is missing or it is wrong then, instead of 2, there can be at most 1 point given for the last item.*
The number of the favourable outcomes is equal to $\binom{5}{3}\cdot\binom{15}{5}$.	1 point	
The probability in question is hence $\frac{\binom{5}{3}\cdot\binom{15}{5}}{\binom{20}{8}}=\frac{10\cdot 3003}{125970}=0.2384$.	2 points	
Total:	**4 points**	

7. a)

wages (in thousand forints)	68	108	154	184	225
no. of employees	25	65	70	44	16
% of employees	**11**	**30**	**32**	**20**	**7**



For the correct chart (proper labelling of the axes is worth 1-1 points each, and the bars are also worth 1 point).

Remark:

The candidate may define the respective heights of the bars either as the number or as the percentage of the employees. There are 11 people corresponding to 5 %. The third row of the table above is not required for the 3 points.

Total:	**3 points**	

b)		
The mean of the August gross income is $\frac{25\cdot 68+65\cdot 108+70\cdot 154+44\cdot 184+16\cdot 225}{220}=\frac{31196}{220}=$ $= 141.8$ thousand Ft.	3 points	*If the mean is correct then detailed computations are not required for the 3 marks. The candidate might have used a calculator where the result is immediately displayed after having keyed in he data.*

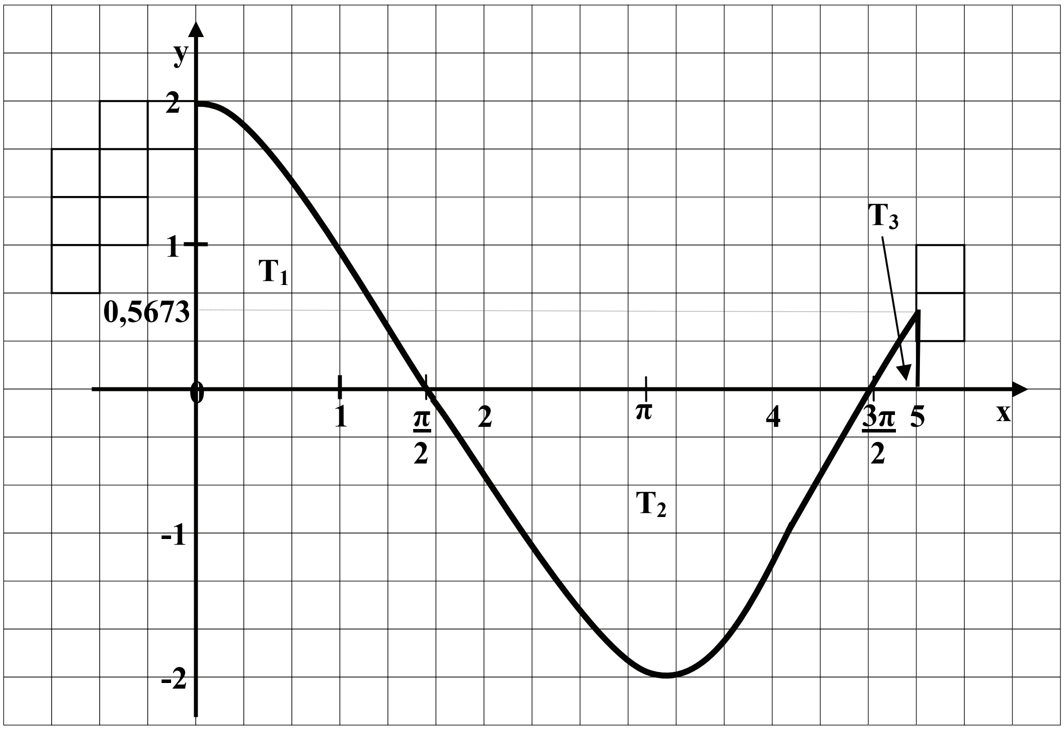
The standard deviation of the August gross income is 43.17 thousand Ft.	3 points	*Any correct calculation of the standard deviation is worth 3 marks. The candidate may proceed from first principles or it can use the „mean of the squares – the square of the mean” formula or a statistical package. If, instead of the standard deviation the candidate announces its square as the answer, at most 2 points can be given.*
Total:	**6 points**	*In case of wrong mean/standard deviation there can be no points given unless it is clear from the paper that the candidate has an accurate knowledge of these notions.*

c)		
The net income of every employee is 60.6% of its gross income.	1 point	*While marking questions c) and d) full score should be given if the candidate finds the correct answer by referring to general results about the mean. If the candidate does not indicate the dimension of the mean and/or standard deviation, (thousand forints, in this case) then its total score on the whole question should be reduced by 1 point*
The payroll of each of the 220 employees is scaled up by 0.606 and thus is the mean. Hence the mean of the net income is $0.606 \cdot 141.8 \approx 85.93$ thousand Ft.	2 points	
Remark: 85.94 thousand Ft may also be accepted as a correct result.		
Total:	**3 points**	
d)		
If the gross income of each of the 220 employee is increased by 2 500 Ft then so is their average gross income.	1 point	
The difference of the new salary and the new mean is the same in each of the 220 cases as that of the precedent values, because both quantities are increased by the same amount, respectively.	1 point	

Accordingly, the mean of the squares of these differences is also invariant: it is equal to the corresponding mean for August.	1 point	
Therefore, the standard deviation of the gross income is not influenced by the promotion.	1 point	
Total:	**4 points**	

8. a)		
The function cos x is odd and thus $f(x) = 2\cos x$	2 points	
The function f is bounded,	1 point	
because $-2 \le f(x) \le 2$.	1 point	
It is not true that both the minimum argument and the maximum value are irrational numbers,	1 point	
because the maximum value of f is 2 which is not irrational.	1 point	
Total:	**6 points**	

b)



A correct sketch includes the marking of the units and also the significant intervals.	2 points	
The region in question is formed by three smaller regions whose area is denoted by T_1, T_2, T_3, respectively. Since the function *f* is continuous, the respective areas can be found by integration: $T_1 = \int_0^{\frac{\pi}{2}} 2\cos x dx = 2\left[\sin x\right]_0^{\frac{\pi}{2}} = 2$.	2 points	*The respective areas T_1 and T_2 can also be found if one takes into account that the areas of the regions enclosed by the graphs of* $2\cos x$ *(or that of* $2\sin x$*) and the* $\left[0\,;\frac{\pi}{2}\right]$ *interval of the x-axis are equal to* 2, *respectively.* *The correct results by themselves are worth 1 point each and also the correct reference in each case.*
The function *f* is not positive on the interval $\left[\frac{\pi}{2};\frac{3\pi}{2}\right]$ and thus $T_2 = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 2\lvert\cos x\rvert dx = 2\left[-\sin x\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = 4$.	2 points	
$T_3 = \int_{\frac{3\pi}{2}}^{5} 2\cos x dx = 2\left[\sin x\right]_{\frac{3\pi}{2}}^{5} = 2\sin 5 + 2$.	2 points	
The area of the region is equal to $T = T_1 + T_2 + T_3 =$ $= 2 + 4 + 2\sin 5 + 2 = 8 + 2\sin 5 \approx 6.082$.	2 points	
Total:	**10 points**	

9.		
First we find the four sets *A*, *B, C* and *D* elementwise that make the statements true, respectively. The elements are two digit numbers. The number *N* is divisible by 7 if it is a multiple of 7. $A = \{ 14; 21; 28; 35; 42; 49; 56; 63; 70; 77; 84; 91; 98 \}$	1 point	
N is a multiple of 29: $B = \{ 29; 58; 87 \}$	1 point	
N + 11 is a square number if $N = n^2 - 11$, where $22 < n^2 < 111$. Therefore, $C = \{ 14; 25; 38; 53; 70; 89 \}$	1 point	
N − 13 is a square number if $N = k^2 + 13$, where $0 \le k^2 < 87$. Therefore, $D = \{ 13; 14; 17; 22; 29; 38; 49; 62; 77; 94 \}$	1 point	
The numbers satisfying the conditions belong to the intersection of some two of the above four sets and, at the same time, they are not elements of the remaining two sets.	1 point	
There are six ways to chose two out of the four sets above. Consider the six intersections hence obtained.	1 point	*The 2 marks for this unit are due even if the candidate does not write down a detailed explanation but the argument clearly shows that it is proceeding along these lines.*
Both $A \cap B$ and $B \cap C$ are empty.	1 point	
$A \cap C = \{ 14; 70 \}$;	1 point	
14 does not belong to *D* and thus it is not a solution; 70, on the other hand is, in fact, a solution, because it does not belong to neither *B*, nor *D*.	1 point	
$A \cap D = \{ 14; 49; 77 \}$;	1 point	
14 belongs to *C* so it is not a solution; both 49 and 77 are solutions because neither of them belongs to any one of *C* and *B*.	1 point	
$B \cap D = \{ 29 \}$,	1 point	
29 is a solution because it is not in *A* neither in *C*.	1 point	
$C \cap D = \{ 14; 38 \}$,	1 point	

14 has already been excluded (it is in *A*, anyway); 38 is a solution since it is neither in *A*, nor in *B*.	1 point	
Coming to the end, there are five numbers altogether satisfying the conditions of the problem: 29; 38; 49; 70; 77.	1 point	
Total:	**16 points**	

Remarks:
If the candidate simply writes down the numbers claimed to be the solutions without actually giving the explanation (not showing the two true statements and the two false ones in each case) then 1 or 2 correct numbers are worth 1 point, 3 correct numbers are worth 2 points, 4 correct numbers are worth 3 points, finally 5 correct numbers are worth 4 points.
If the candidate proves that his numbers are indeed solutions but does not exclude any other number then at most 8 points can be given. (The double of the respective scores of the previous list.)